Curve Fitting Example

Example 8: Find the cubic polynomial

$$f(t) = x_1 + x_2 t + x_3 t^2 + x_4 t^3 \tag{1}$$

that satisfies $f(-2) = 3$, $f(-1) = -6$, $f(1) = 0$, and $f(3) = -2$. Verify your answer.

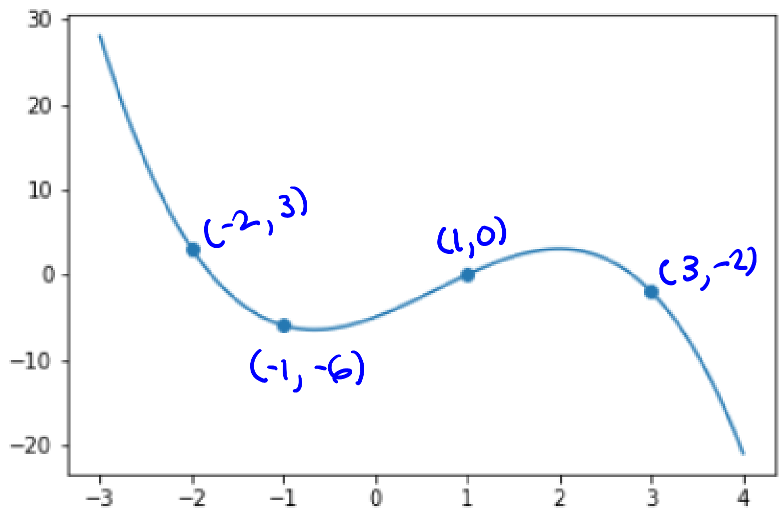

Figure 1: Graph of cubic polynomial passing through $(-2, 3)$, $(-1, -6)$, $(1, 0)$, and $(3, -2)$

$$
\begin{aligned}
f(-2)=3 &\Rightarrow x_1 + (-2)x_2 + (-2)^2 x_3 + (-2)^3 x_4 = 3\\
f(-1)=-6 &\Rightarrow x_1 + (-1)x_2 + (-1)^2 x_3 + (-1)^3 \cdot x_4 = -6\\
f(1)=0 &\Rightarrow x_1 + (1)x_2 + (1)^2 x_3 + (1)^3 x_4 = 0\\
f(3)=-2 &\Rightarrow x_1 + 3x_2 + (3)^2 x_3 + (3)^3 x_4 = -2
\end{aligned}
$$

$$
\begin{cases}
x_1 - 2x_2 + 4x_3 - 8x_4 = 3\\
x_1 - x_2 + x_3 - x_4 = -6\\
x_1 + x_2 + x_3 + x_4 = 0\\
x_1 + 3x_2 + 9x_3 + 27x_4 = -2
\end{cases}
\qquad
\left[\begin{array}{cccc|c}
1 & -2 & 4 & -8 & 3\\
1 & -1 & 1 & -1 & -6\\
1 & 1 & 1 & 1 & 0\\
1 & 3 & 9 & 27 & -2
\end{array}\right] \sim
$$

Extra Space:

$R_4 := R_4 - R_1$
$R_3 := R_3 - R_1$
$R_2 := R_2 - R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 3 \\ 0 & 1 & -3 & 7 & -9 \\ 0 & 3 & -3 & 9 & -3 \\ 0 & 5 & 5 & 35 & -5 \end{array}\right]$$

$R_4 := R_4 - 5R_2$
$R_3 := R_3 - 3R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 3 \\ 0 & 1 & -3 & 7 & -9 \\ 0 & 0 & 6 & -12 & 24 \\ 0 & 0 & 20 & 0 & 40 \end{array}\right]$$

$R_4 := \frac{1}{20} R_4$
$R_3 := \frac{1}{6} R_3$

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 3 \\ 0 & 1 & -3 & 7 & -9 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right]$$

$R_4 := R_4 - R_3$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 3 \\ 0 & 1 & -3 & 7 & -9 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 2 & -2 \end{array}\right]$$

$\Rightarrow x_2 - 3(2) + 7(-1) = -9 \Rightarrow x_2 = 4$
$\Rightarrow x_3 - 2(-1) = 4 \Rightarrow x_3 = 2$
$\Rightarrow 2x_4 = -2 \Rightarrow x_4 = -1$

$x_1 - 2(4) + 4(2) - 8(-1) = 3$
$x_1 = 8 - 8 - 8 + 3 \Rightarrow x_1 = -5$

$$\boxed{f(t) = -5 + 4t + 2t^2 - t^3}$$

Check: $f(-2) = -5 + 4(-2) + 2(-2)^2 - (-2)^3 = 3$ ✓

$f(-1) = -5 + 4(-1) + 2(-1)^2 - (-1)^3 = -6$ ✓

$f(1) = -5 + 4(1) + 2(1)^2 - (1)^3 = 0$ ✓

$f(3) = -5 + (3) + 2(3)^2 - (3)^3 =$

$= -5 + 12 + 18 - 27 = -2$ ✓